

Turbulent Comptonization in Relativistic Accretion Disks

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Abstract

Turbulent Comptonization, a potentially important damping and radiation mechanism in relativistic accretion flows, is discussed. Particular emphasis is placed on the physical basis, relative importance, and thermodynamics of turbulent Comptonization. The effects of metal-absorption opacity on the spectral component resulting from turbulent Comptonization is considered as well.

1. Overview of Thermal and Bulk Comptonization

If a beam of photons interacts with a scattering layer consisting of relatively hot electrons such that $k_B T_e \gg h\nu$, the input seed photons gain energy while conserving their overall number, a process known as Compton up-scattering or “Comptonization” [7]. In the non-relativistic limit, the shift in photon energy per scattering is small, so that $\Delta E/E \sim k_B T_e / m_e c^2$. The rare photons that remain in the scattering layer the longest are subsequently boosted to energies limited by the electron temperature – a process that is statistically identical to the classical theory of cosmic ray acceleration [3] – and the resulting spectral energy distribution resembles a power law.

The fractional shift in energy per scattering event is proportional to the mean square electron velocity i.e., $\Delta E/E \sim \langle v_e^2 \rangle / c^2$ where $\langle \rangle$ is taken over the electron distribution function. Also, if the scattering layer moves in bulk with respect to

the observer, then the radiation will experience a Doppler shift such that $\Delta E/E \sim v/c$, an effect often referred to in cosmological contexts as the “kinetic Sunyaev- Zeldovich effect.” However, if the scattering layer is turbulent and stationary with respect to the observer, then the Doppler shift term that is $\propto v/c$ vanishes despite the presence of large scale bulk motions. It follows that the contribution to Comptonization from turbulence leads to an amplification $\Delta E/E \sim v_T^2/c^2$, where v_T is the turbulent velocity. This effect, which we refer to as “turbulent Comptonization,” deforms the spectra of soft photons in a manner indistinguishable from that of thermal Comptonization [6],[10].²

In this note, we discuss certain aspects of turbulent Comptonization in the context of relativistic accretion disk theory. Many of the results summarized here can be found in [8].

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² In the language of cosmology, turbulent Comptonization might be referred to as the “second-order kinetic Sunyaev-Zeldovich effect.”

2. Presence of Turbulent Comptonization in Relativistic Accretion Flows

Turbulent Comptonization dominates over thermal Comptonization when $v_T^2 > v_{th}^2$, where v_{th} is the electron thermal velocity. In what follows, we briefly outline why it is possible to satisfy this condition in relativistic accretion flows.

Consider a thin accretion disk extending down to the innermost stable circular orbit around a rapidly rotating black hole [5], [9]. The orbital velocity near its inner-edge approaches the speed of light. For accretion rates close to the Eddington limit, the inner-edge of the disk becomes marginally thick i.e., the disk scale height H is comparable to its radius R from the black hole. As a result, the sound speed of the disk c_s near the central object approaches the speed of light since the thermal content of the disk is close to the virial value. In this parameter regime, thin relativistic accretion disks are radiation pressure dominated, potentially allowing for the radiation sound speed to exceed the electron thermal velocity. By assuming that the accretion stress $\tau_{R\phi}$ both scales with the total pressure and is turbulent in nature [1], we conclude that *in principle*, the turbulent velocity $v_T > v_{th}$ close to the central gravitating object. Therefore, it is possible for turbulent Comptonization, rather than thermal Comptonization to be the radiation mechanism responsible for the radiative release of gravitational energy. Note that the above arguments are *independent of black hole (or neutron star) mass*.

The radiative emission of both Galactic X-ray binaries (XRBs) and active galactic nuclei (AGNs) are most commonly modeled with a multi-temperature disk component, accompanied by a Comptonizing corona. Generally speaking, the physics of two-phased systems that are composed of a relatively cool, dense, optically thick, turbulent layer which releases a fraction of its binding energy in a hot diffuse optically thin corona is not well-understood. A good example of a system that illustrates our lack of predictive power is the solar chromosphere and corona. In other words, there remains a great deal of uncertainty in identifying the physical mechanisms responsible for converting dissipated gravitational binding energy into

radiative power.

Provided that an adequate supply of soft seed photons is available, the spectral component arising from turbulent Comptonization does not suffer from the same level of theoretical uncertainty as one modeled from a multi-temperature disk + corona. Specifically, the physical mechanism responsible for the accretion is also directly responsible for at least a portion of the radiative energy release.

3. Transfer Effects and the y -parameter

At this preliminary level of accuracy, we model accretion disk turbulence as a non-linear cascade of energy down to small scales, where the outer driving scale of the turbulence is occupied and powered by relatively large-scale modes driven by the magnetorotational instability (MRI) [1]. Radiation pressure supported thin accretion flow onto relativistic objects are optically thick to Thomson scattering. In a one-zone height-averaged disk model, the photon mean free path λ_p is likely to be small in comparison to the outer scale of an MRI cascade. If this is the case, photons cannot “sample” the velocities of the most powerful eddies.

If, for example, accretion disk turbulence follows an isotropic Kolmogorov scaling, the velocity amplitude of the turbulence scales as $v_T(\lambda) \sim v_T(\lambda_0)(\lambda/\lambda_0)^{1/3}$, implying that

$$T_T(\lambda) \sim T_T(\lambda_0) \left(\frac{\lambda}{\lambda_0} \right)^{2/3}. \quad (1)$$

Here, λ_0 represents the outer scale of the turbulence and $T_T \equiv 3m_e v_T^2 / 2k_B$ can be thought of as a turbulent electron temperature. On scales where $\lambda = \lambda_p$, seed photons can efficiently extract energy from the turbulence as they are being boosted by it.

The arguments presented in §1 suggest that the spectral deformation arising from turbulent Comptonization is identical to that of thermal Comptonization. For example, in the unsaturated case, the radiation spectrum due to a single homogeneous scattering layer is represented by a cut-off temperature, y -parameter, and normalization. In

§2, we showed that the outer scale turbulent velocity, and therefore temperature, of thin accretion flows is independent of the mass of the central object. Likewise, the turbulent y -parameter on the outer scale is given by

$$y_T \sim \frac{4k_B T_T (\lambda_0)}{m_e c^2} \tau^2, \quad (2)$$

which for thin disks, is *independent of mass and radius* near the inner-edge of the disk [8], where a great fraction of the binding energy is radiated.

Perhaps it is not surprising that the Compton temperature of XRBs and AGNs are comparable in value, despite the enormous range in central mass. That is, Compton temperatures typical to both systems hover close to ~ 100 keV – the electron virial temperature a few gravitational radii away from the surface of the black hole. The Compton y -parameter for XRBs and AGN are quite similar as well, implying that the fraction of energy dissipated per unit area above the depth of formation of the cool optically thick component is comparable in the two cases. If turbulent Comptonization is responsible for the broad band X-ray emission of relativistic accretion flows, then the arguments surrounding eq. (2) might explain why the spectral shape is similar in the two classes of sources.

4. Compton Cooling of Accretion Disk Turbulence

An important check of the internal consistency of our previous arguments is to make sure that the rate of energy injected into the turbulent cascade is equal to the rate of energy gain by the radiation field. A version of the Kompaneets equation that takes into account bulk motions [4],[10],[6] allows one to estimate the turbulent Compton cooling rate

$$t_{C,T}^{-1} \sim \sigma_{es} n_e c \frac{v_T^2}{3c^2} = \sigma_{es} n_e c \frac{k_B T_T}{m_e c^2} \quad (3)$$

where σ_{es} and n_e is the electron scattering cross section and the electron number density, respectively. The above expression can also be interpreted as the approximate rate at which turbulence is

“cooled” or damped by the radiation field. For thin disks, Socrates et al. [8] point out that

$$t_{C,T}^{-1} \sim \frac{\tau}{c} \frac{\alpha P}{H \rho} \sim \frac{\alpha \tau}{c} \Omega^2 H \sim \Omega \quad (4)$$

where Ω , α , P , and ρ is the Keplerian frequency, the thin disk viscosity parameter, mid-plane pressure, and density, respectively.

If accretion disk turbulence is driven by the MRI, then the rate at which energy is injected into the turbulent cascade i.e., the eddy-turnover time $\sim \Omega$. Remarkably, the rate of energy injection into the random turbulent motions is comparable to the rate at which the radiation field extracts energy from them. An implication that stems from eq. (4) is that turbulent Comptonization may serve as an important source of dissipation for accretion disk turbulence, while also functioning as a radiation mechanism which mediates the release of binding energy.

5. Ionization Balance and Metal Edge Opacity

In order to amplify seed photons, Comptonizing turbulence must be bound within a Thomson scattering layer, where the effective optical depth to absorption is less than unity. Near the inner-edge, the disk is likely to be optically thin, but effectively thick to free-free emission. In this region, the site of formation is close to the mid-plane, implying that continuum absorption is negligible in the above adjacent scattering layer, for a significant number of scattering depths [8].

Though it is straightforward to argue that radiation pressure supported accretion disks are effectively thin to free-free absorption down to significant depth, the notion that the same holds true for sources of bound-free absorption is not so easily grasped. Again, the key issue circles around whether or not it is more probable for a seed photon to be up-scattered to X-ray energies or to be absorbed by, in this case, a C,N,O, or Fe atom. To answer this question, we estimate the optical depth to absorption from metal edges.

In order to obtain a rough idea of how important metal edges are, we work with a one-zone model. Also, for the sake of simplicity, we consider the case of saturated turbulent Comptonization, where the energy of all of the disk photons are roughly given by $k_B T_T$ (λ_p). We further assume that every bound electron lies in a hydrogenic state of some fiducial ion of abundance A_Z .

The above simplifications allow for an estimation of the hydrogen-like metal ionization fraction by equating the photo-ionization rate to the recombination rate i.e.,

$$n_i \int_{\nu_0}^{\infty} d\nu \frac{E_\nu \sigma_\nu}{h\nu} c \simeq n_i n_{\gamma,I} \sigma_0 c = n_e n_{i+1} \alpha_R(T), \quad (5)$$

where n_i , n_{i+1} , n_e , σ_0 , and α_R is the number density of bound ions, the number density of stripped ions, the number density of liberated electrons, the photo-ionization cross section at the threshold photon frequency ν_0 , and the recombination rate, respectively. Also, the number density of ionizing photons near the midplane $n_{\gamma,I}$ is roughly given by $n_{\gamma,I} \sim P_r / k_B T_T$, where P_r is the radiation pressure at the disk midplane.

The ionization fraction $\chi_i \equiv n_i / n_{i+1}$ is given by

$$\chi_i \simeq \frac{n_e \alpha_R(T)}{n_{\gamma,I} \sigma_0 c} \sim \frac{T_T}{T_d} \frac{P_g}{P_r} \frac{\alpha_R(T)}{\sigma_0 c}. \quad (6)$$

Here, P_g is the midplane gas pressure. This allows us to compare the optical depth resulting from bound-free absorption τ_{abs} to that of Thomson scattering τ_{es}

$$\frac{\tau_{abs}}{\tau_{es}} \sim \frac{A_Z \chi_i \sigma_0}{\sigma_{es}} \sim A_Z \frac{T_T}{T_d} \frac{P_g}{P_r} \frac{\alpha_R(T)}{\sigma_{es} c} \sim 3 \times 10^{-3} \quad (7)$$

for $A_Z = 10^{-3}$, $T_T / T_d = 10^2$, $P_g / P_r = 10^{-5}$, and $\alpha_R(T) = 10^{-10}$. Note that $\tau_{es} \sim (\alpha \dot{m})^{-1}$ for thin disks, implying that $\tau_{eff} \sim \sqrt{\tau_{abs} \tau_{es}} \lesssim 1$ if $\alpha \sim 0.1$ and $\dot{m} \lesssim \dot{m}_{Edd}$. Thus, it seems plausible that photons amplified by Comptonizing turbulence can survive long enough to escape, before being absorbed by the metal content of the disk.

6. Discussion

The mechanisms that result in the randomization of gravitational binding energy, transport of angular momentum, and the conversion of binding energy into heat and radiation must be simultaneously understood if a *predictive* theory of accretion is desired. It is widely believed that MRI turbulence is responsible for the randomization of gravitational binding energy and the transport of angular momentum[1]. Also, sophisticated techniques used to model the spectra of relativistic accretion flows fit the data quite well. Nevertheless, there is currently no accepted method of connecting the underlying MHD to the observable radiation spectrum, implying that accretion theory suffers from an overall lack of predictive power. We argue that turbulent Comptonization alleviates some of these inadequacies even if it is not the dominant mechanism of radiative energy release, since it allows observers to peer into the inner workings of the turbulence which is thought to be the prime mover of the accretion process.

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